Hamiltonian dynamics for sampling

```
Var burglary = sample(Bernoulli(0.1))
```

```
Var burglary = sample(Bernoulli(0.1))

Var somethingDifficult = sample(Complex(1,3))
```

What to do now?

- Enumeration
- Rejection sampling
- Importance sampling
- MCMC
- Particle filtering

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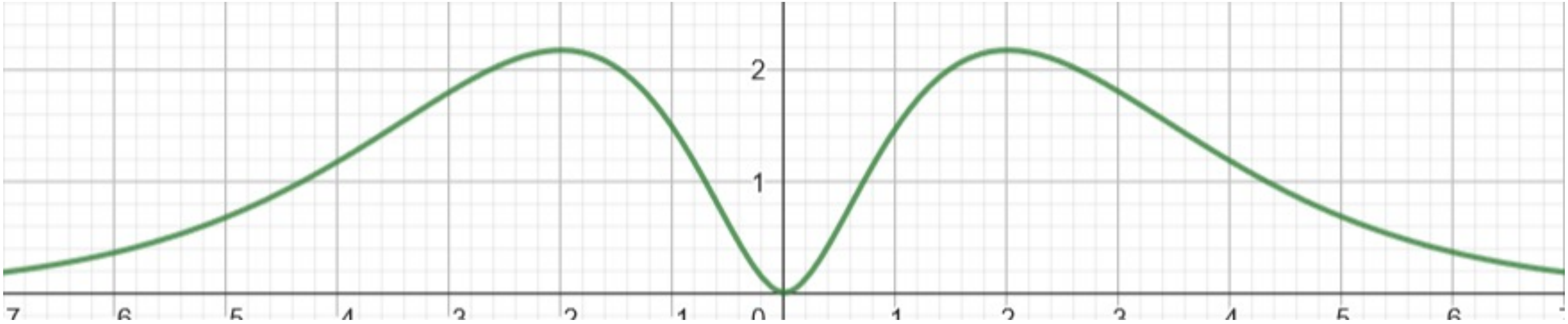

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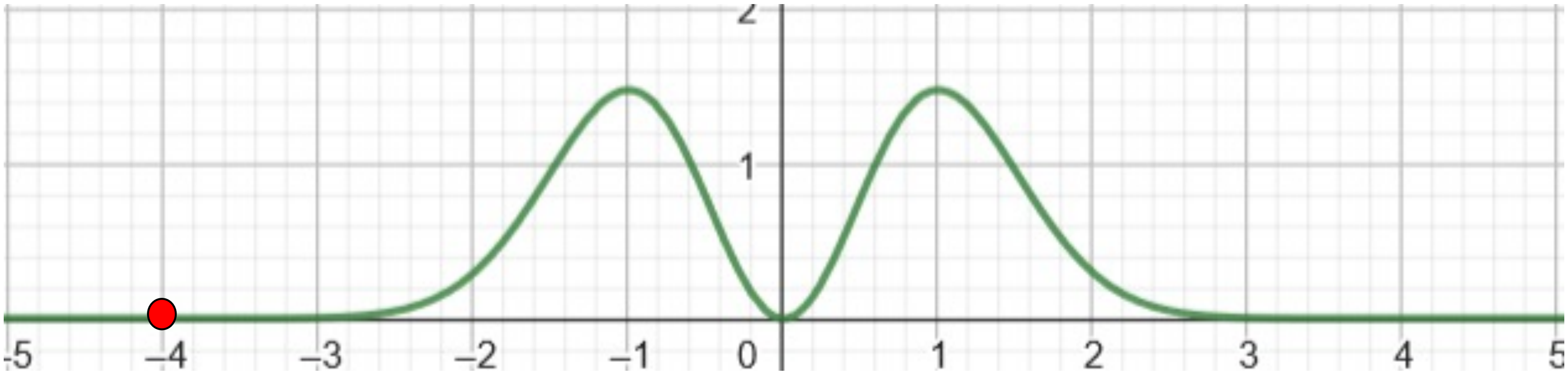

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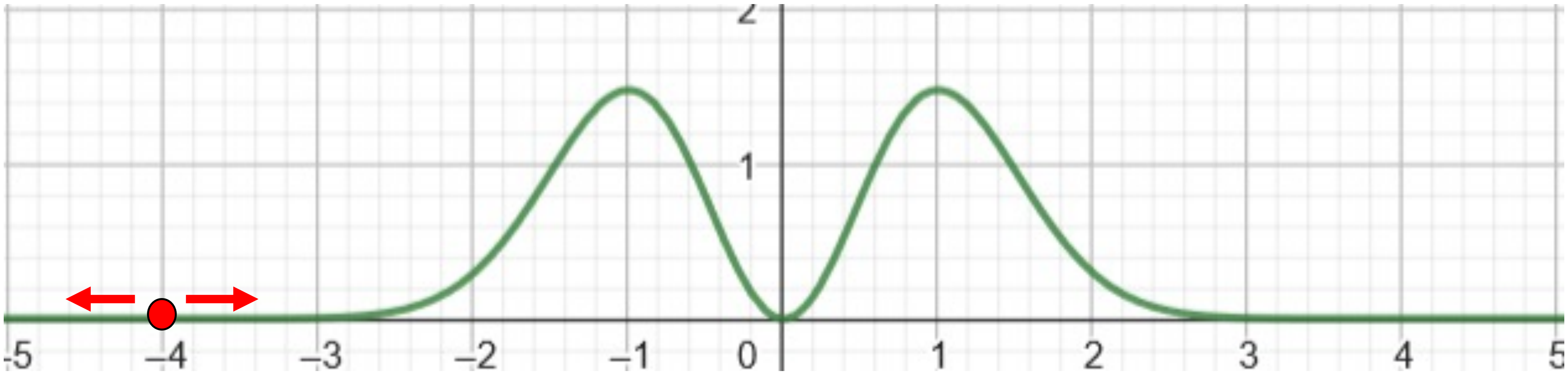

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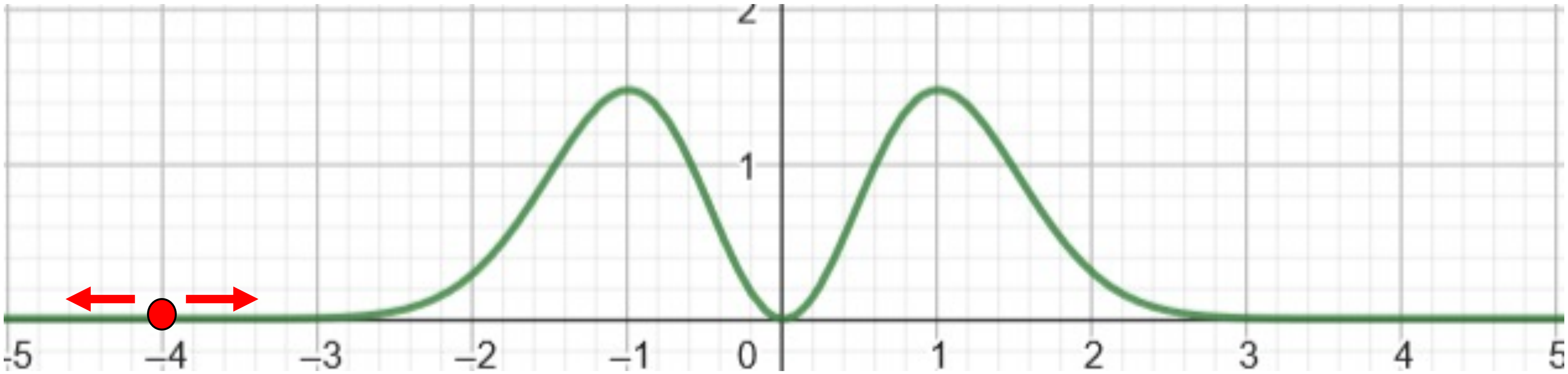
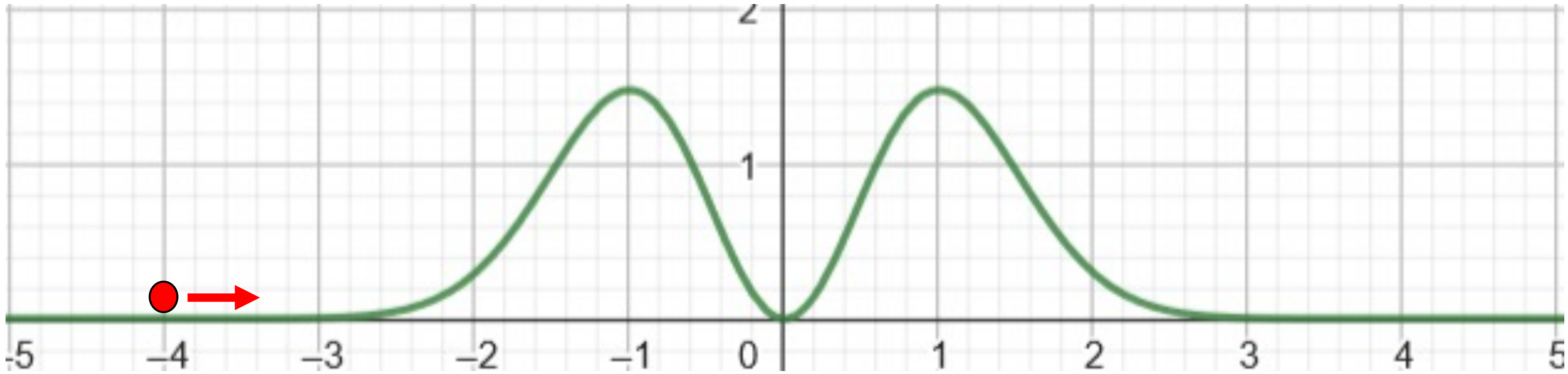


Try left and right, with equal probability
- Will find the left peak eventually
- Will find the right peak when given even more time

What does HMC do?



Some physics: Hamiltonian

We take: $H=U+K$

With, usually, $K=p^2$

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$$\frac{dp}{dt} = -\frac{dH}{dq}$$

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Some physics: Hamiltonian

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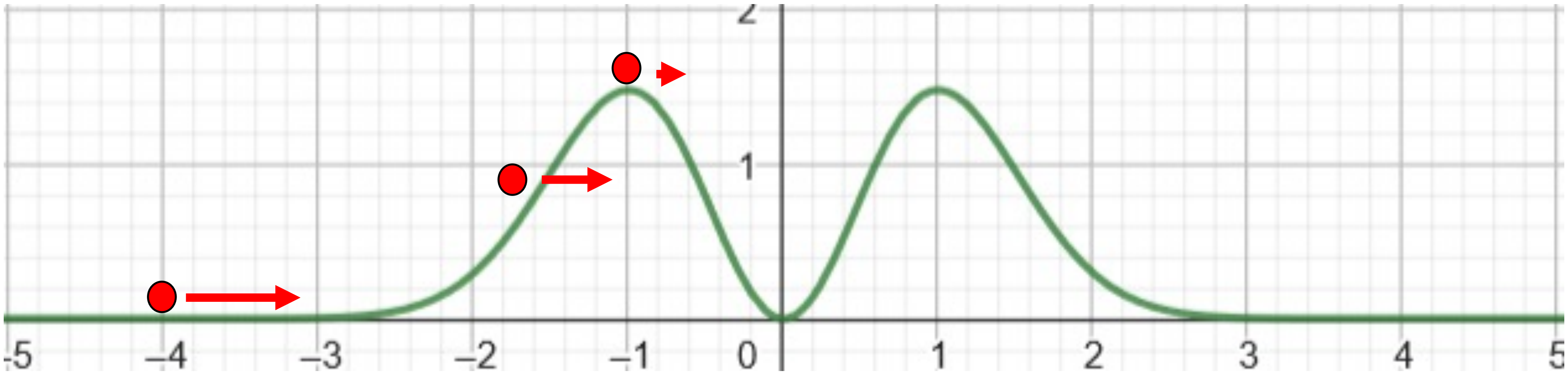
With, usually, $K=p^2$

1. Conservation of energy, the Hamiltonian
2. Reversibility
3. Volume conservation

What does HMC do?



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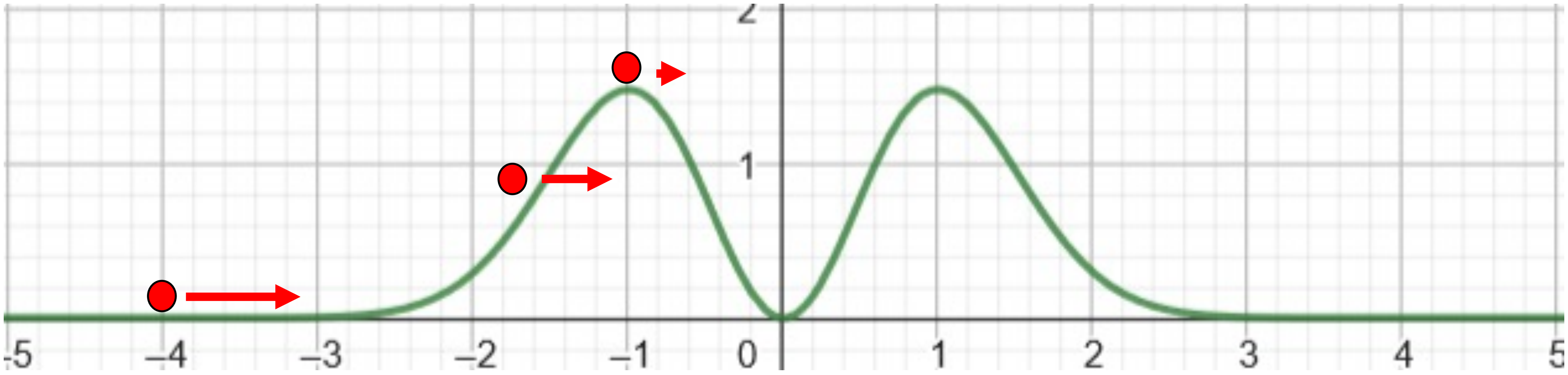
- We use U for our target, U=-ln(f(q))

$$P(q,p) = \frac{f(q)}{Z}\exp\left(-\frac{p^2}{2}\right)$$

More technical



We start with some state and momentum
We do *L* leapfrog integration steps according to Hamiltonian dynamics

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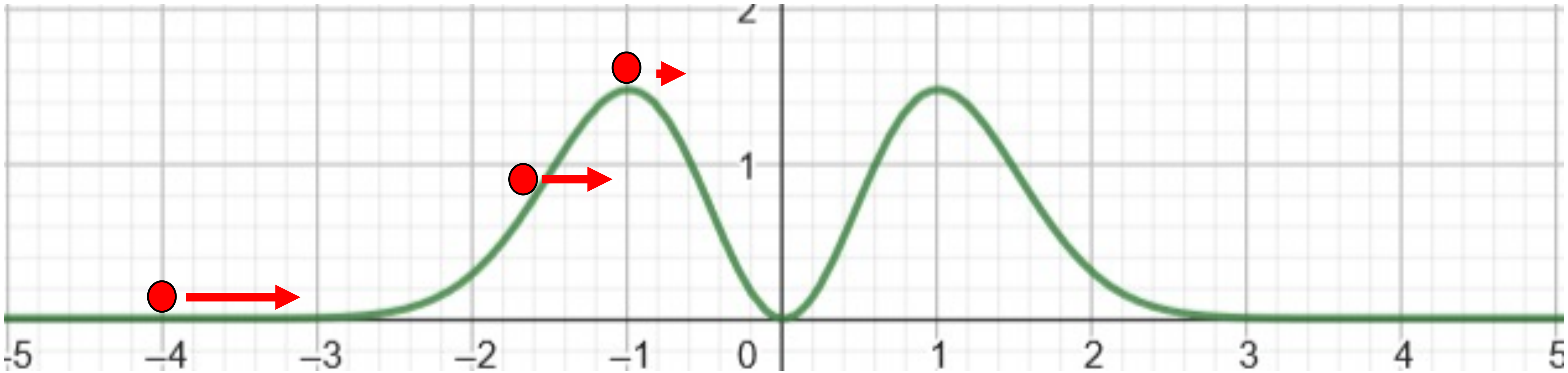

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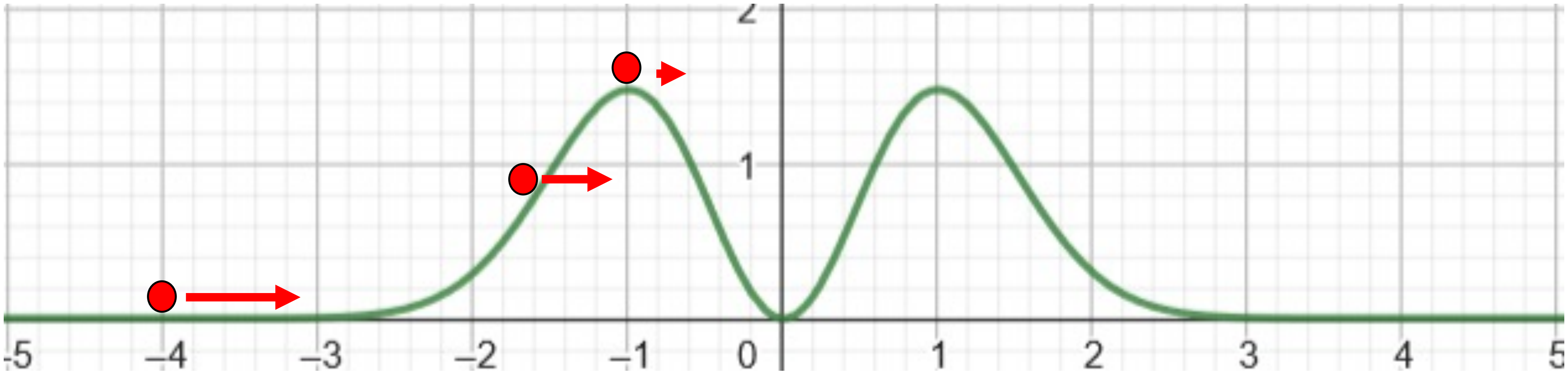

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Thus: we skip a slow random walk

Some example performance

$$H(q,p) \;=\; q^T\Sigma^{-1}q\,/\,2 \;+\; p^Tp\,/\,2, \qquad \text{with } \Sigma = \begin{bmatrix} 1 & 0.95 \\ 0.95 & 1 \end{bmatrix}$$

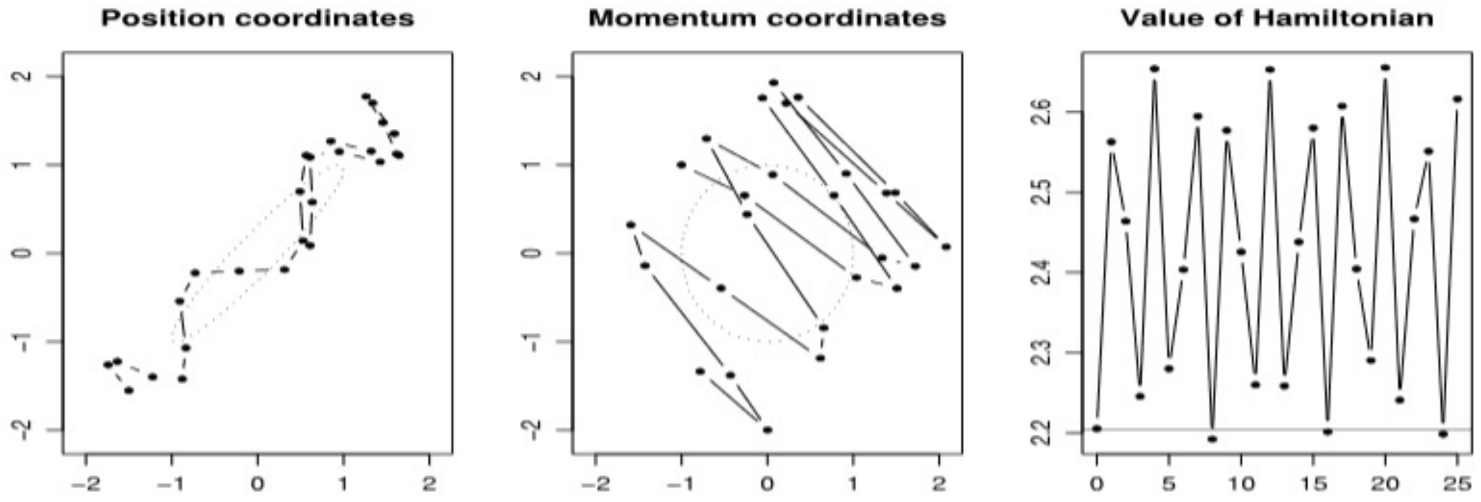

Figure 3: A trajectory for a 2D Gaussian distribution, simulated using 25 leapfrog steps with a stepsize of 0.25. The ellipses plotted are one standard deviation from the means. The initial state had $q = [-1.50, -1.55]^T$ and $p = [-1, 1]^T$.